Measurement uncertainty evaluation of the load loss of power transformers

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1 Summary

In this example, the evaluation of measurement uncertainty of power transformer losses are given. It is shown how this evaluation can be performed using the law of propagation of uncertainty from the "Guide to the expression of uncertainty in measurement" (GUM) as well as the propagation of distributions using the Monte Carlo method from GUM Supplement 1 (GUM-S1). It is shown how the approach using the Monte Carlo method can be used to validate the output from the law of propagation of uncertainty from the GUM.

2 Introduction of the application

When it comes to electrical energy production, transformation, distribution or consumption, energy losses generated by transformers play a very important role. These losses form the second largest part of the total losses in the distribution of energy and the network. Measurement results of power transformer losses should be accurate as these are often the object of guarantee and penalty in many contracts and hence play an important role in billing. Costs of losses in power transformers are comparable to the product cost and play an important role in the evaluation of the total costs of obtaining the energy at the point of use.

Many European regulations set requirements for reliable loss values. For instance, the Ecodesign Directive [1] provides EU rules for improving environmental performance of products, among which power transformers are recognized. The directive sets out minimum mandatory requirements for the energy efficiency of these products, which helps to prevent creation of barriers to trade, improve product quality and yield environmental protection. The European Commission Regulation [2] on implementation of Directive [1] with regard to small, medium and large power transformers recognised energy in the use phase as one of the most significant environmental aspect that can be addressed through product design. This regulation provides basic measurements concepts as well as tolerances to be achieved in order to comply with this regulation.

In the case of market surveillance, if a power transformer facility exceeds the guaranteed limit for losses, a fine must be paid. Therefore, IEC 60076-19 [3] recommends that the evaluated uncertainty should be less than the tolerance limit set out in regulations. The measurement uncertainty is often not taken into account in considering the agreement between the manufacturer and costumer on who will pay the fine in case the losses exceed the guaranteed value. Furthermore, IEC 60076-19 recommends that guarantee and penalty calculations should refer to the best estimated values of the losses without considering the measurement uncertainties, based on a shared risk concept, where both parties are aware of and accept the consequences of non-negligible measurement uncertainty.

In this example, it is shown how the evaluation of the measurement uncertainty for load losses can be conducted to improve existing practice for power load loss measurements. Two approaches are used: the law of propagation of uncertainty of the GUM [4] and the propagation of distributions [5]. The latter method is applied to validate the uncertainty evaluation method proposed in IEC 60076-19 [3].

3 Specification of the measurand(s)

The measurand in this example is power load loss, stated at the rated current at a given reference temperature. A value for the measurand is either obtained by measuring the power load loss at the rated current, or by recalculation to this current. The measurand is specified at a temperature. The conversion to that reference temperature is part of the measurement model.

4 Measurement model

The total load loss P_{LL} of a transformer at reference temperature θ_{r}, summed over the three phases, can be expressed as (adapted from [3])[1]

$$P_{\mathrm{LL}} = \sum_{i=1}^{3} I_{\mathrm{NHV}}^2 R_{1,\mathrm{HV},i} \frac{t+\theta_{\mathrm{r}}}{t+\theta_1} + I_{\mathrm{NLV}}^2 R_{1,\mathrm{LV},i} \frac{t+\theta_{\mathrm{r}}}{t+\theta_1} + P_{\mathrm{a2},i} \frac{t+\theta_2}{t+\theta_{\mathrm{r}}}, \tag{1}$$

where I_{NHV} denotes the rated primary phase current, I_{NLV} the rated secondary phase current, $R_{1,\mathrm{HV},i}$ the resistance of the primary winding at temperature θ_1 for phase i, $R_{1,\mathrm{LV},i}$ the resistance of the secondary winding at temperature θ_1 for phase i, t a constant for the winding material, θ_1 the temperature during the resistance measurement, θ_2 the temperature during loss measurement, θ_{r} the reference temperature and $P_{\mathrm{a2},i}$ the additional loss for phase i. The index i runs over the three phases (denoted by U, V, and W). The additional loss is computed as

$$P_{\mathrm{a2},i} = P_{2,i} - I_{\mathrm{NHV},i}^2 R_{2,\mathrm{HV},i} - I_{\mathrm{NLV},i}^2 R_{2,\mathrm{LV},i}, \tag{2}$$

where

$$R_2 = R_1 \frac{t+\theta_2}{t+\theta_1}, \tag{3}$$

[1] IEC 60076-19 does not provide an explicit equation for the summation over the three phases of the alternating current. In this example, we provide this expression explicitly, and substitute for the summands (the load loss $P_{\mathrm{LL},i}$ of the three phases the expression given in the said standard instead).

and the power $P_{2,i}$ measured at the load loss measurement corrected for known systematic deviations and referred to the current $I_{\mathrm{N,prim},i}$ is computed as

$$P_{2,i} = k_{\mathrm{CN}}(1+\varepsilon_{\mathrm{C}})k_{\mathrm{VN}}\frac{1}{1+\varepsilon_{\mathrm{V}}}\frac{P_{\mathrm{W},i}}{1-(\Delta_{\varphi V,i}-\Delta_{\varphi C,i})\tan\varphi_i}\left(\frac{I_{\mathrm{NHV},i}}{k_{\mathrm{CN}}I_{\mathrm{M},i}}\right)^2, \tag{4}$$

where

$\left(\frac{I_{\mathrm{NHV}}}{k_{\mathrm{CN}}I_{\mathrm{M},i}}\right)^2$	the term related to the actual current $I_{\mathrm{M},i}$, rated to the reference current I_{NHV} for which transformer shall be tested,
k_{CN}	rated transformation ratio of the current transformer,
k_{VN}	rated transformation ratio of the voltage transformer,
ε_{C}	actual ratio error of the current transformer (% of nominal ratio),
ε_{V}	actual ratio error of the voltage transformer (% of nominal ratio),
$\frac{1}{1-(\Delta_{\varphi\mathrm{V},i}-\Delta_{\varphi\mathrm{C},i})\tan\varphi_i}$	the term related to the correction for phase displacement ($F_{\mathrm{D},i}$) of the current ($\Delta_{\varphi\mathrm{C},i}$) and voltage transformers ($\Delta_{\varphi\mathrm{V},i}$),
$P_{\mathrm{W},i}$	reading of the wattmeter and the wattmeter is considered to have unknown errors, and thus no correction term.

The actual phase angle φ_i between voltage and current under the sinusoidal conditions normally valid for load-loss measurement depends on the phase angle measured with power meter (φ_{M}) and is obtained from

$$\varphi_i = \varphi_{\mathrm{M},i} - (\Delta_{\varphi\mathrm{V},i}-\Delta_{\varphi\mathrm{C},i}) = \arccos\left(\frac{P_{\mathrm{W},i}}{I_{\mathrm{M},i}U_{\mathrm{M},i}}\right) - (\Delta_{\varphi\mathrm{V},i}-\Delta_{\varphi\mathrm{C},i}). \tag{5}$$

The substitution of equations (2) and (3) into (1) yields

$$\begin{aligned}
P_{\mathrm{LL}} &= \sum_{i=1}^{3} I_{\mathrm{NHV}}^2 R_{1,\mathrm{HV},i}\frac{t+\theta_{\mathrm{r}}}{t+\theta_1} + I_{\mathrm{NLV}}^2 R_{1,\mathrm{LV},i}\frac{t+\theta_{\mathrm{r}}}{t+\theta_1} + \left(P_{2,i} - I_{\mathrm{NHV},i}^2 R_{2,\mathrm{HV},i} - I_{\mathrm{NLV},i}^2 R_{2,\mathrm{LV},i}\right)\frac{t+\theta_2}{t+\theta_{\mathrm{r}}} \\
&= \frac{t+\theta_{\mathrm{r}}}{t+\theta_1}\sum_{i=1}^{3}\left(I_{\mathrm{NHV}}^2 R_{1,\mathrm{HV},i} + I_{\mathrm{NLV}}^2 R_{1,\mathrm{LV},i}\right) + \frac{t+\theta_2}{t+\theta_{\mathrm{r}}}\sum_{i=1}^{3}\left(P_{2,i} - I_{\mathrm{NHV},i}^2 R_{2,\mathrm{HV},i} - I_{\mathrm{NLV},i}^2 R_{2,\mathrm{LV},i}\right) \\
&= \left(\frac{t+\theta_{\mathrm{r}}}{t+\theta_1} - \frac{t+\theta_2}{t+\theta_{\mathrm{r}}}\frac{t+\theta_2}{t+\theta_1}\right)\sum_{i=1}^{3}\left(I_{\mathrm{NHV}}^2 R_{1,\mathrm{HV},i} + I_{\mathrm{NLV}}^2 R_{1,\mathrm{LV},i}\right) + \frac{t+\theta_2}{t+\theta_{\mathrm{r}}}\sum_{i=1}^{3} P_{2,i}
\end{aligned} \tag{6}$$

5 Evaluation of the input quantities

5.1 Specifications and measurement data

The characteristics of the transformer are summarised in table 1. The rated primary and secundary currents are considered constants, that is, without uncertainty. The same applies to the reference temperature θ_{r}. As the windings are made from copper (Cu), $t = 235$ and also treated as a constant [3].

Table 1: Specification of the phase transformer – dry type transformer [3]

Component	Symbol	Value	Unit
Rated power	S_r	630	kV A
Rated primary voltage	U_{NHV}	6000	V
Rated secondary voltage	U_{NLV}	400	V
Rated primary phase current	I_{NHV}	60.62	A
Rated secondary phase current	I_{NLV}	909.33	A
Reference temperature	θ_r	120	°C
Winding material		Cu	

This example uses the specifications of the transformer as well as of the measuring instruments. These specifications are summarised in table 2.

Table 2: Specifications of measuring system current and voltage transducers for the three phases U, V and W (adapted from IEC 60076-19 [3])

Component	Symbol/Unit	U	V	W
CT accuracy class		0.2	0.2	0.2
CT ratio	k_{CN}	1	1	1
CT max. amplitude error	$e_{class,CT}/\%$	±0.2	±0.2	±0.2
CT max. phase displacement	$\Delta_{\phi,C}/'$	±10	±10	±10
VT accuracy class		0.2	0.2	0.2
VT ratio	k_{VN}	1	1	1
VT max. amplitude error	$e_{class,VT}/\%$	±0.2	±0.2	±0.2
VT max. phase displacement	$\Delta_{\phi,V}/'$	±10	±10	±10
Maximum of current range of Power Meter	I_{max}/A	50	50	50
Maximum of voltage range of Power Meter	U_{max}/V	240	240	240

The measured data are summarised in table 3.

Table 3: Experimental data of the load loss measurement [3]

Component	Symbol/Unit	U	V	W
Resistance HV	R_{HV}/Ω	0.0490	0.0500	0.0510
Resistance LV	R_{LV}/Ω	0.001 600	0.001 500	0.001 700
Temp. during resistance meas.	$\theta_1/°C$	22.1	22.1	22.1
Voltage Phase-Neutral	U_M/V	191.5	195.2	194.8
Phase current	I_M/A	40.55	40.2	40.6
Measured active power	P_W/W	748	756	762
Power factor	$\cos\varphi/1$	0.096 33	0.096 34	0.096 35
Temp. during loss meas.	$\theta_2/°C$	21.8	21.8	21.8

Starting with the input quantities in equation (4), ε_C is modelled as having a rectangular distribution (uniform distribution) with zero mean and semi-width $e_{class,CT}$ (see table 2). The standard uncertainty is given by

$$u(\varepsilon_{\rm C}) = \frac{e_{\rm class,CT}}{\sqrt{3}}.$$

In the application of the law of propagation of uncertainty of the GUM, the fact will be exploited that equation (4) can be considered as a product of a set of independent quantities of the kind

$$Y = cX_1^{p_1}X_2^{p_2}\ldots X_N^{p_N},$$

where c denotes a constant. If the input quantities $X_1, X_2, \ldots, X_N$ are mutually independent, then the squared relative standard uncertainty associated with the measurand Y can be expressed as follows in terms of the relative standard uncertainties associated with the input quantities [4, clause 5.1.6]:

$$u_{\rm rel}^2(Y) = \sum_{i=1}^{N} p_i^2 u_{\rm rel}^2(X_i)$$

Considering $1+\varepsilon_{\rm C}$ as a term in the product, the relative standard uncertainty of this term is needed and is given by

$$u_{\rm rel}(1+\varepsilon_{\rm C}) = \frac{u(\varepsilon_{\rm C})}{1+\varepsilon_{\rm C}}. \tag{7}$$

The evaluation of the standard uncertainty of $\varepsilon_{\rm V}$ is very similar to that of $\varepsilon_{\rm C}$. $\varepsilon_{\rm V}$ is modelled using the rectangular distribution with zero mean and semi-width $e_{\rm class,VT}$. The relative standard uncertainty of $1/(1+\varepsilon_{\rm V})$ is given by

$$u_{\rm rel}\left(\frac{1}{1+\varepsilon_{\rm V}}\right) = \frac{u(\varepsilon_{\rm V})}{1+\varepsilon_{\rm V}} \tag{8}$$

using the fact that $|u_{\rm rel}(1/X)| = |u_{\rm rel}(X)|$.

The standard uncertainty of the power measurement $P_{{\rm W},i}$ is based on the specification for the maximum permissible error. This error is specified as $0.015\,\%\,P_{{\rm W},i} + 0.010\,\%\,P_{\rm max}$ where $P_{\rm max} = I_{\rm max}U_{\rm max} = 12\,000\,{\rm W}$. $P_{{\rm W},i}$ is modelled using the rectangular distribution with mean $P_{{\rm W},i}$ and semi-width the error thus computed.

The standard uncertainty associated with the phase displacement $F_{{\rm D},i}$ according to IEC 60076-19 [3] is evaluated as follows. The relative standard uncertainty is approximated by

$$u_{\rm rel}(F_{{\rm D},i}) = \tan\varphi_i\, u(\Delta_{\varphi V,i} - \Delta_{\varphi {\rm C},i}), \tag{9}$$

where [3]

$$u(\Delta_{\varphi V,i} - \Delta_{\varphi {\rm C},i}) = \sqrt{u^2(\Delta_{\varphi V,i}) + u^2(\Delta_{\varphi {\rm C},i})}.$$

The standard uncertainties $u(\Delta_{\varphi V,i})$ and $u(\Delta_{\varphi C,i})$ are obtained from the rectangular distribution with zero mean and the maximum errors specified in table 2.

In the uncertainty evaluation described in IEC 60076-19 [3], the uncertainty associated with φ_i is not considered. When applying the law of propagation of uncertainty, the sensitivity coefficient equals $\Delta_{\varphi V,i} - \Delta_{\varphi {\rm C},i}$ and is zero in this particular case (the values of $\Delta_{\varphi V,i}$ and $\Delta_{\varphi {\rm C},i}$ are both zero).

The standard uncertainty associated with φ_i can be obtained from equation (5). The derivative of $\arccos x$ is $-1/\sqrt{1-x^2}$ [6] and that of $\tan x$ is $\cos^{-2} x$ [6]. So,

$$u^2(\tan\varphi_i) \approx \frac{1}{\cos^4\varphi_i} u^2(\varphi_i)$$

and

$$u^2(\varphi_i) \approx \frac{1}{1-[P_W/(I_{\mathrm{M},i}U_{\mathrm{M},i}))]^2} u^2\left(\frac{P_{\mathrm{W},i}}{I_{\mathrm{M},i}U_{\mathrm{M},i}}\right) + u^2(\Delta_{\varphi\mathrm{C},i}) + u^2(\Delta_{\varphi\mathrm{V},i})$$

and

$$u^2_{\mathrm{rel}}\left(\frac{P_{\mathrm{W},i}}{I_{\mathrm{M},i}U_{\mathrm{M},i}}\right) = u^2_{\mathrm{rel}}(P_{\mathrm{W},i}) + u^2_{\mathrm{rel}}(I_{\mathrm{M},i}) + u^2_{\mathrm{rel}}(U_{\mathrm{M},i}),$$

which are obtained using the law of propagation of uncertainty from the GUM [4]. Given that both $\tan x$ and $\arccos x$ are strongly non-linear functions, the results are only approximate; hence the $\approx$ sign. Strictly speaking, this applies also to all expressions for the relative standard uncertainty based on clause 5.1.6 of the GUM.

The uncertainty associated with the current $I_{\mathrm{M},i}$ is evaluated as follows. The specification of the current measurement is that the maximum permissible error equals $0.01\,\%\,I_{\mathrm{M},i} + 0.02\,\%\,I_{\max}$. The current is modelled using the rectangular distribution with centred at $I_{\mathrm{M},i}$ with semi-width equal to the maximum permissible error.

The standard uncertainty of the voltage measurement $U_{\mathrm{M},i}$ is based on the specification for the maximum permissible error. This error is specified as $0.010\,\%\,U_{\mathrm{M},i} + 0.018\,\%\,U_{\max}$. $U_{\mathrm{M},i}$ is modelled using the rectangular distribution centred at $U_{\mathrm{M},i}$ with semi-width equal to the error thus computed.

Use of the specifications provides the relative standard uncertainties in table 4. This uncertainty component can be included in the uncertainty budget by approximating the sensitivity coefficient numerically using the definition of a derivative. In that case, the sensitivity coefficient is 1 (the partial derivatives are respectively $\partial\Delta_{\varphi\mathrm{C},i}/\partial\Delta_{\varphi\mathrm{C},i}$ and $\partial\Delta_{\varphi\mathrm{V},i}/\partial\Delta_{\varphi\mathrm{V},i}$).

Table 4: Uncertainty evaluation of $\tan\varphi_i$ for the three phases based on equation (5). Values in percentages are relative standard uncertainties; other values are absolute standard uncertainties

Source	Unit	Phase		
		U	V	W
CT max. phase displacement	(rad)	0.001 679	0.001 679	0.001 679
VT max. phase displacement	(rad)	0.001 679	0.001 679	0.001 679
Phase current unc	%	0.020	0.020	0.020
Measured active power unc	%	0.101	0.100	0.100
Voltage phase-neutral unc	%	0.019	0.019	0.019
$u(\cos\varphi)$		0.002 377	0.002 377	0.002 377
$u(\tan\varphi)$		0.26	0.26	0.26

6 Propagation of uncertainty

6.1 Law of propagation of uncertainty

The propagation of uncertainty for the power measured at load loss P_2 (equation (4)) is performed using the formula for a measurement model as a pure product (see GUM clause 5.16 [4]). Substituting the values and specifications provides the following relative uncertainties (table 5) for the three phases.

Table 5: Uncertainty evaluation for the measured load loss P_2 at ambient temperature (adapted from [3])

Quantity	Component	$u_{\mathrm{rel}}/\%$			Exponent
		U	V	W	
CT ratio error	ε_{C}	0.115	0.115	0.115	1
VT ratio error	ε_{V}	0.115	0.115	0.115	1
Measured power	P_{W}	0.101	0.100	0.100	1
Phase displacement	F_{D}	2.454	2.454	2.454	1
Ampere meter	I_{M}	0.040	0.040	0.040	2
Load loss	P_2	2.930	2.928	2.926	

The values of the measured and additional load loss obtained according to equations (1) and (2) are shown in table 6.

Table 6: Values of measured load loss and additional load loss at temperature θ_2 and load losses at temperature θ_r

Quantity	Component	Value /W		
		U	V	W
Measured load loss at θ_2	P_2	1671.7	1719.1	1698.8
Additional load loss	P_{a2}	170.4	296.7	107.5
I_{N}^2R loss	I_{N}^2R	1501.3	1422.4	1591.3
Load loss at θ_r	P_{LL}	2198.7	2181.0	2277.5

It should be noted that both the primary and secondary current and resistance of the windings are taken into account when calculating additional load loss.

The sensitivity coefficients for calculation of the uncertainty contributions of the uncertainty of resistance at temperature θ_2 are computed in accordance with equation (3) as

$$\frac{\partial R_2}{\partial R_1} = \frac{t+\theta_2}{t+\theta_1}, \tag{10}$$

$$\frac{\partial R_2}{\partial \theta_1} = -\frac{R_2}{t+\theta_1}, \tag{11}$$

$$\frac{\partial R_2}{\partial \theta_2} = \frac{R_2}{t+\theta_2}. \tag{12}$$

The sensitivity coefficient as the result from the mathematical formulation (2) of the measured load loss at ambient temperature equals 1, whereas its standard uncertainty is obtained from the relative standard uncertainties given in table 5. The sensitivity coefficient for the resistance of the windings at temperature θ_2 is equal to the squared rated current for both the primary and secondary coil. Both quantities, temperature during resistance measurement and load loss measurement, have rectangular distributions with mean 0 K and semi-width 1 K. The uncertainty contributions of each component are given in table 7.

Table 7: Uncertainty of additional load loss at temperature θ_2

Quantity	Component	Uncertainty contribution /W (sensitivity coeff. × standard unc.)		
		U	V	W
Resistance HV at θ_1	$R_{1,\mathrm{HV}}$	2.83×10^{-5}	2.88×10^{-5}	2.94×10^{-5}
Resistance LV at θ_1	$R_{1,\mathrm{LV}}$	9.23×10^{-7}	8.65×10^{-7}	9.80×10^{-7}
Temp. resistance meas. (HV)	θ_1	-1.09×10^{-4}	-1.12×10^{-4}	-1.14×10^{-4}
Temp. resistance meas. (LV)	θ_1	-3.59×10^{-6}	-3.36×10^{-6}	-3.81×10^{-6}
Temp. load loss meas. (HV)	θ_2	1.10×10^{-4}	1.12×10^{-4}	1.14×10^{-4}
Temp. load loss meas. (LV)	θ_2	3.59×10^{-6}	3.37×10^{-6}	3.82×10^{-6}
Measured load loss at θ_2	P_2	2398.91	2534.03	2471.27
Resistance HV at θ_2	$R_{2,\mathrm{HV}}$	0.58	0.59	0.60
Resistance LV at θ_2	$R_{2,\mathrm{HV}}$	4.27	4.00	4.53
Additional load loss at θ_2	$u(P_{\mathrm{a2}})$	49.17	50.50	49.92

Using equation (1) and the computed values given in the table 6 the uncertainty of load loss at reference temperature θ_r is calculated for all three phases. The values in table 8 are obtained by taking into account the sensitivity coefficient for each quantity in the measurement model (1), except the reference temperature and indicated currents which are considered to be constant.

Table 8: Uncertainty of load losses reported at reference temperature θ_r

Quantity	Component	Uncertainty /W		
		U/W	V/W	W/W
Reported load loss	$u(P_{\mathrm{LL}})$	35.89	36.81	36.46
Expanded uncertainty of reported load loss ($k = 2$)	$U(P_{\mathrm{LL}})$	71.77	73.63	72.93

The total load loss, computed as the load loss of the three phases U, V, and W, is 6657 W with standard uncertainty 63 W.

6.2 Monte Carlo method

The implementation of the propagation of distributions is carried using the measurement model as given in equations (1), (2), (4) and (5). As to the input quantities, the same probability density functions are used as for the evaluation using the law of propagation of uncertainty.

For the purpose of describing the R software used for the evaluation, the variables holding the measured values and some constants are declared below, where their names are largely self-explanatory (table 9. Finally, the measured values for resistance, power, current and voltage are declared similarly.

Table 9: Cross reference of symbols and variable names used in the R code

Variable	Symbol	Variable in R code
Offset for winding material	t	tW
Rated maximum current	I_{max}	I_max
Rated maximum voltage	U_{max}	U_max
Rated maximum power	P_{max}	P_max
Temperature during resistance measurement	θ_1	theta1.val
Temperature during load loss measurement	θ_2	theta2.val
Reference temperature	θ_{ref}	theta_r
CT ratio	k_{CN}	k_CN
VT ratio	k_{VN}	k_VN
CT maximum amplitude relative error	ε_{C}	eps_C.val
VT maximum amplitude relative error	ε_{V}	eps_V.val
Half-width of the rectangular distribution of ε_{C}		eps_C.hw
Half-width of the rectangular distribution of ε_{V}		eps_V.hw
Currents in the high-voltage and low-voltage circuit	I_{NHV}	texttt I_NHV
Currents in the high-voltage and low-voltage circuit	I_{NLV}	I_NLV

```
t_W = 235
I_max = 50
U_max = 240
P_max = I_max*U_max

theta1.val = 22.1 # rectangular distribution, half-width = 1 K
theta2.val = 21.8 # rectangular distribution, half-width = 1 K
theta_r = 120
k_CN = 1
k_VN = 1
eps_C.val = 0.0 # rectangular distribution, half-width = 0.2% of k_CN
eps_C.hw = 0.002
eps_V.val = 0.0 # rectangular distribution, half-width = 0.2% of k_VN
eps_V.hw = 0.002
I_NHV = 60.62
I_NLV = 909.33
Delta_C = 0.0 # rectangular distribution, half-width = 10 arcmin
Delta_C.hw = 10/(60*180)*pi
Delta_V = 0.0 # rectangular distribution, half-width = 10 arcmin
Delta_V.hw = 10/(60*180)*pi
```

```
# measured resistances for the three phases
R_HV = c(0.049,0.0500,0.0510)
# at theta1 ... (rectangular, half-width 0.1% relative)
R_LV = c(0.001600,0.001500,0.001700)
# at theta1 ... (rectangular, half-width 0.1% relative)

# measured power, current and voltage for the three phases
P_W = c(748,756,762) # rectangular distribution
I_M = c(40.55,40.2,40.6) # rectangular distribution
U_M = c(191.5,195.2,194.8) # rectangular distribution
```

The three functions needed to calculate the load loss are coded as follows (see code below). The constants are declared as variables with a default value in the function heading, which enables calling the function without having to include these variables (R will then use their default values). This approach makes the code less cluttered.

```
# power P_2
P2func <- function(k_CN=1,eps_C=0,k_VN=1,eps_V=0,P_W,Delta_C=0,Delta_V=0,
I_M,U_M,I_N) {
phi = acos(P_W/(I_M*U_M)) - (Delta_V-Delta_C)
k_CN*(1/(1+eps_C))*k_VN*1/(1+eps_V)*P_W/
(1-(Delta_V-Delta_C)*tan(phi))*(I_N/(k_CN*I_M))^2
}

# temperature correction
temp.corr <- function(R,theta1,theta2,t_W = 235) {
R*(t_W+theta2)/(t_W+theta1)
}

# load loss for a single phase
PLL.func <- function(I_NHV,I_NLV,R_2HV,R_2LV,P2,theta2,theta_r = theta_r,
t_W = 235) {
I_NHV^2*temp.corr(R_2HV,theta2 = theta_r,theta1 = theta2)+I_NLV^2*
temp.corr(R_2LV,theta2 = theta_r,theta1 = theta2) +
(P2-I_NHV^2*R_2HV-I_NLV^2*R_2LV)*(t_W+theta2)/(t_W+theta_r)
}
```

To implement the Monte Carlo method, the R function `runif()`, which provides a random variate with a rectangular distribution, is recast so that it can be called with a central value and half-width. The number of trials M is set to 10^6. First, the variables are generated for all phases, followed by generating the data for the phases U, V, and W. The phase angle `phi` (ϕ) is calculated as described in IEC 60076-19 [3].

The measurement model in this application is quite involved in that it contains several steps. Also, it should be noted that the summation over the three phases is not necessarily a summation of three independent quantities, that is, the load losses for the phases U, V, and W are correlated. All these aspects are taken care of by the Monte Carlo method, as outlined below.

```
rrect <- function(num,middle=0,hw=1) {
runif(n=num,min=middle-hw,middle+hw)
}
```

```r
# MC data
M = 1000000

# all phases
theta1 = rrect(M,theta1.val,1.0)
theta2 = rrect(M,theta2.val,1.0)
eps_C = rrect(M,eps_C.val,eps_C.hw)
eps_V = rrect(M,eps_V.val,eps_V.hw)

# phase U
Delta_C.U = rrect(M,0.0,10*pi/(60*180)) # use specified half-width
Delta_V.U = rrect(M,0.0,10*pi/(60*180))
R_HV.U = rrect(M,R_HV[1],0.001*R_HV[1])
R_LV.U = rrect(M,R_LV[1],0.001*R_LV[1])
U_M.U = rrect(M,U_M[1],0.00010*U_M[1]+0.00018*U_max)
I_M.U = rrect(M,I_M[1],0.00010*I_M[1]+0.00020*I_max)
P_W.U = rrect(M,P_W[1],0.00015*P_W[1]+0.00010*P_max)

R_2HV.U = temp.corr(R_HV.U,theta1 = theta1,theta2 = theta2)
R_2LV.U = temp.corr(R_LV.U,theta1 = theta1,theta2 = theta2)

phi = acos(P_W.U/(I_M.U*U_M.U)) - (Delta_V.U-Delta_C.U)
F_D.U = 1/(1-(Delta_V.U-Delta_C.U)*tan(phi))

P2.U = P2func(eps_C = eps_C, eps_V = eps_V,P_W = P_W.U,Delta_C = Delta_C.U,
Delta_V = Delta_V.U,I_M = I_M.U,U_M = U_M.U,I_N = I_NHV)

PLL.U = PLL.func(I_NHV = I_NHV,I_NLV = I_NLV,R_2HV = R_2HV.U,R_2LV = R_2LV.U,
P2 = P2.U,theta2 = theta2, theta_r = theta_r)

# phase V
Delta_C.V = rrect(M,0.0,10*pi/(60*180)) # use specified half-width
Delta_V.V = rrect(M,0.0,10*pi/(60*180))
R_HV.V = rrect(M,R_HV[2],0.001*R_HV[2])
R_LV.V = rrect(M,R_LV[2],0.001*R_LV[2])
U_M.V = rrect(M,U_M[2],0.00010*U_M[2]+0.00018*U_max)
I_M.V = rrect(M,I_M[2],0.00010*I_M[2]+0.00020*I_max)
P_W.V = rrect(M,P_W[2],0.00015*P_W[2]+0.00010*P_max)

phi = acos(P_W.V/(I_M.V*U_M.V)) - (Delta_V.V-Delta_C.V)
F_D.V = 1/(1-(Delta_V.V-Delta_C.V)*tan(phi))

P2.V = P2func(eps_C = eps_C, eps_V = eps_V,P_W = P_W.V,Delta_C = Delta_C.V,
Delta_V= Delta_V.V,I_M = I_M.V,U_M = U_M.V,I_N = I_NHV)

R_2HV.V = temp.corr(R_HV.V,theta1 = theta1,theta2 = theta2)
R_2LV.V = temp.corr(R_LV.V,theta1 = theta1,theta2 = theta2)

PLL.V = PLL.func(I_NHV = I_NHV,I_NLV = I_NLV,R_2HV = R_2HV.V,R_2LV = R_2LV.V,
P2 = P2.V,theta2 = theta2, theta_r = theta_r)
```

```r
# phase W
Delta_C.W = rrect(M,0.0,10*pi/(60*180)) # use specified half-width
Delta_V.W = rrect(M,0.0,10*pi/(60*180))
R_HV.W = rrect(M,R_HV[3],0.001*R_HV[3])
R_LV.W = rrect(M,R_LV[3],0.001*R_LV[3])
U_M.W = rrect(M,U_M[3],0.00010*U_M[3]+0.00018*U_max)
I_M.W = rrect(M,I_M[3],0.00010*I_M[3]+0.00020*I_max)
P_W.W = rrect(M,P_W[3],0.00015*P_W[3]+0.00010*P_max)

R_2HV.W = temp.corr(R_HV.W,theta1 = theta1,theta2 = theta2)
R_2LV.W = temp.corr(R_LV.W,theta1 = theta1,theta2 = theta2)

phi = acos(P_W.W/(I_M.W*U_M.W)) - (Delta_V.W-Delta_C.W)
F_D.W = 1/(1-(Delta_V.W-Delta_C.W)*tan(phi))

P2.W = P2func(eps_C = eps_C, eps_V = eps_V,P_W = P_W.W,Delta_C = Delta_C.W,
Delta_V = Delta_V.W,I_M = I_M.W,U_M = U_M.W,I_N = I_NHV)


PLL.W = PLL.func(I_NHV = I_NHV,I_NLV = I_NLV,R_2HV = R_2HV.W,R_2LV = R_2LV.W,
P2 = P2.W,theta2 = theta2, theta_r = theta_r)

# total load loss
PLL.tot = PLL.U + PLL.V + PLL.W

#output
# c(mean(PLL.tot),sd(PLL.tot),quantile(PLL.tot,probs=c(0.025,0.975)))
PLL.tot.val = mean(PLL.tot)
PLL.tot.unc = sd(PLL.tot)
PLL.tot.Unc = 0.5*(quantile(PLL.tot,probs=c(0.975))-
quantile(PLL.tot,probs=c(0.025)))
PLL.tot.k = PLL.tot.Unc/PLL.tot.unc
```

The load loss is 6657 W with standard uncertainty 53 W, about 15 % smaller than that obtained with the GUM uncertainty framework. The expanded uncertainty is obtained using the `quantile()` function in R as shown above. The coverage factor is obtained by dividing the expanded uncertainty by the standard uncertainty. The expanded uncertainty is 104 W with a coverage factor $k = 1.95$. The relevant guidance can be found in GUM-S1 [5, clause 7.6] and GUM-S2 [7, clause 7.6].

The assessment of the strength of the dependencies between the measurement results of the three phases U, V, and W can be readily accomplished in R, using the output of the Monte Carlo method. First, the vectors `PLL.U`, `PLL.V` and `PLL.W` are combined into a matrix called `PLL.matrix`. Then, using the R-function `cor`, the correlation matrix is obtained:

```r
PLL.matrix = cbind(PLL.U,PLL.V,PLL.W)
cor(PLL.matrix)

##             PLL.U      PLL.V      PLL.W
## PLL.U 1.00000000 0.01333439 0.01479030
## PLL.V 0.01333439 1.00000000 0.01474804
## PLL.W 0.01479030 0.01474804 1.00000000
```

From the correlation matrix thus obtained, it is evident that the correlation coefficients between phases are very small (all positive and less than 0.015, compared with unity for fully correlated values). Hence, summing the results of the three phases as if they were independent (uncorrelated) is justified.

7 Reporting the result

According to the Monte Carlo method, the load loss is 6657 W with standard uncertainty 53 W. The expanded uncertainty is 104 W and the coverage factor, computed as the ratio of the expanded uncertainty and standard uncertainty is 1.95.

The GUM uncertainty framework gives the same estimate, namely 6657 W, as the Monte Carlo method and a somewhat larger standard uncertainty of 63 W. Assuming a normal distribution yields for 95 % coverage probability a coverage factor of 1.96, and thus an expanded uncertainty of 124 W.

8 Interpretation of results

The outcomes of the GUM uncertainty framework and the Monte Carlo method agree quite well. The Monte Carlo method enables, as demonstrated, to assess the validity of the assumption made when applying the law of propagation of uncertainty that the results of the three phases can be treated as independent.

An improvement of the treatment could be the consideration that the results of the three phases are dependent (due to using the same equipment for the measurement of the electrical quantities of the three phases).

For the data used, the Monte Carlo method, regarded by many (such as [8–12]) as a 'gold standard' for uncertainty propagation, indeed validates the GUM results although the latter can be regarded as moderately conservative. In this instance, the Monte Carlo method is also used to justify treating the losses over the three phases as independent (see also equation (1)).

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References

[1] Directive 2009/125/EC of the European Parliament and of the Council on establishing a framework for the setting of ecodesign requirements for energy-related products, 2009.

[2] Comission Regulation (EU) 548/2014 on implementing Directive 2009/125/EC of the European Parliament and of the Council with regard to small, medium and large power transformers, 2014.

[3] IEC 60076-19 Power transformers – Part 19: Rules for the determination of uncertainties in the measurement of the losses on power transformers and reactors. IEC, International Electrotechnical Commission, Geneva, Switzerland, 2013.

[4] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML. *Guide to the Expression of Uncertainty in Measurement, JCGM 100:2008, GUM 1995 with minor corrections*. BIPM, 2008.

[5] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML. *Supplement 1 to the 'Guide to the Expression of Uncertainty in Measurement' – Propagation of distributions using a Monte Carlo method, JCGM 101:2008*. BIPM, 2008.

[6] R. C. Weast. *CRC Handbook of Chemistry and Physics: A Ready-Reference Book of Chemical and Physical Data*. CRC Press Inc, 64th edition, 1984.

[7] BIPM, IEC, IFCC, ILAC, ISO, IUPAC, IUPAP, and OIML. *Supplement 2 to the 'Guide to the Expression of Uncertainty in Measurement' – Extension to any number of output quantities, JCGM 102:2011*. BIPM, 2011.

[8] M. G. Cox and A. O'Hagan. Meaningful expressions of uncertainty in measurement. Technical Report MS 27, National Physical Laboratory, 2021.

[9] D. Hangleiter, I. Roth, D. Nagaj, and J. Eisert. Easing the Monte Carlo sign problem. *Science Advances*, 6(33):eabb8341, 2020.

[10] J. H. Huggins, T. Campbell, M. Kasprzak, and T. Broderick. Practical bounds on the error of Bayesian posterior approximations: A nonasymptotic approach. *arXiv e-prints*, page arXiv:1809.09505, Sep 2018.

[11] V. Periyasamy and M. Pramanik. Advances in Monte Carlo Simulation for Light Propagation in Tissue. *IEEE Reviews in Biomedical Engineering*, 10:122–135, 2017.

[12] J. Saini, E. Traneus, D. Maes, R. Regmi, S. R. Bowen, C. Bloch, and T. Wong. Advanced Proton Beam Dosimetry Part I: review and performance evaluation of dose calculation algorithms. *Translational Lung Cancer Research*, 7(2):171–179, 2018.